Chapter 7  
Bonds and Their Valuation  

Solutions to End-of-Chapter Problems

7-1  With your financial calculator, enter the following:

N = 10; I/YR = YTM = 9%; PMT = 0.08 \times 1,000 = 80; FV = 1000; PV = V_B = ?  
PV = $935.82.

7-2  V_B = $985; M = $1,000; Int = 0.07 \times $1,000 = $70.

a.  N = 10; PV = -985; PMT = 70; FV = 1000; YTM = ?  
Solve for I/YR = YTM = 7.2157% = 7.22%.

b.  N = 7; I/YR = 7.2157; PMT = 70; FV = 1000; PV = ?  
Solve for V_B = PV = $988.46.

7-3  The problem asks you to find the price of a bond, given the following facts:  N = 2 \times 8 = 16; I/YR = 8.5/2 = 4.25; PMT = (0.09/2) \times 1,000 = 45; FV = 1000.

With a financial calculator, solve for PV = $1,028.60.

7-4  With your financial calculator, enter the following to find YTM:

N = 10 \times 2 = 20; PV = -1100; PMT = 0.08/2 \times 1,000 = 40; FV = 1000; I/YR = YTM = ?  
YTM = 3.31% \times 2 = 6.62%.

With your financial calculator, enter the following to find YTC:

N = 5 \times 2 = 10; PV = -1100; PMT = 0.08/2 \times 1,000 = 40; FV = 1050; I/YR = YTC = ?  
YTC = 3.24% \times 2 = 6.49%.

Since the YTC is less than the YTM, investors would expect the bonds to be called and to earn the YTC.

7-5  
a.  5%:  Bond L:  Input N = 15, I/YR = 5, PMT = 100, FV = 1000, PV = ?, PV = $1,518.98.  
Bond S:  Change N = 1, PV = ? PV = $1,047.62.

2.  8%:  Bond L:  From Bond S inputs, change N = 15 and I/YR = 8, PV = ?, PV = $1,171.19.  
Bond S:  Change N = 1, PV = ? PV = $1,018.52.

3.  12%:  Bond L:  From Bond S inputs, change N = 15 and I/YR = 12, PV = ?, PV = $863.78.  
Bond S:  Change N = 1, PV = ? PV = $982.14.
b. Think about a bond that matures in one month. Its present value is influenced primarily by the maturity value, which will be received in only one month. Even if interest rates double, the price of the bond will still be close to $1,000. A 1-year bond’s value would fluctuate more than the one-month bond’s value because of the difference in the timing of receipts. However, its value would still be fairly close to $1,000 even if interest rates doubled. A long-term bond paying semiannual coupons, on the other hand, will be dominated by distant receipts, receipts that are multiplied by \(1/(1 + r_d/2)^t\), and if \(r_d\) increases, these multipliers will decrease significantly. Another way to view this problem is from an opportunity point of view. A 1-month bond can be reinvested at the new rate very quickly, and hence the opportunity to invest at this new rate is not lost; however, the long-term bond locks in subnormal returns for a long period of time.

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<thead>
<tr>
<th>Time</th>
<th>Years to Maturity</th>
<th>Price of Bond C</th>
<th>Price of Bond Z</th>
</tr>
</thead>
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<tr>
<td>t = 0</td>
<td>4</td>
<td>$1,012.79</td>
<td>$693.04</td>
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<td>759.57</td>
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<td>t = 2</td>
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<td>832.49</td>
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<td>t = 3</td>
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<tr>
<td>t = 4</td>
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7-7

<table>
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<th>Bond Price Paths</th>
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<td>Years Remaining to Maturity</td>
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<td>4</td>
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<tr>
<td>2</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>0</td>
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</table>

7-7

<table>
<thead>
<tr>
<th>Time</th>
<th>Years to Maturity</th>
<th>Price of Bond C</th>
<th>Price of Bond Z</th>
<th>Percentage Change</th>
</tr>
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<td>1,000.00</td>
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</table>

7-8

The rate of return is approximately 15.03%, found with a calculator using the following inputs:

\[N = 6; PV = -1000; PMT = 140; FV = 1090; I/YR = ?\] Solve for \(I/YR = 15.03\%\).

Despite a 15% return on the bonds, investors are not likely to be happy that they were called. Because if the bonds have been called, this indicates that interest rates have fallen sufficiently that the YTC is less than the YTM. (Since they were originally sold at par, the YTM at issuance = 14%.)
Rates are sufficiently low to justify the call. Now investors must reinvest their funds in a much lower interest rate environment.

7-9  
\[ V_B = \sum_{t=1}^{N} \frac{\text{INT}}{(1 + r_c)^t} + \frac{M}{(1 + r_c)^N} \]

M = $1,000. PMT = 0.09($1,000) = $90.

1. \( V_B = \$829 \): Input \( N = 4, PV = -829, PMT = 90, FV = 1000, YTM = I/YR = ? \) \( I/YR = 14.99\% \).

2. \( V_B = \$1,104 \): Change \( PV = -1104, YTM = I/YR = ? \) \( I/YR = 6.00\% \).

b. Yes. At a price of \$829, the yield to maturity, 15\%, is greater than your required rate of return of 12\%. If your required rate of return were 12\%, you should be willing to buy the bond at any price below \$908.88.

7-10  
\[ N = 9, PV = -901.40, PMT = 80, FV = 1000 \]
\[ I/YR = YTM = 9.6911\% \]

b. The current yield is defined as the annual coupon payment divided by the current price.
\[ CY = \frac{80}{901.40} = 8.875\% \]

Expected capital gains yield can be found as the difference between YTM and the current yield.
\[ CGY = YTM - CY = 9.691\% - 8.875\% = 0.816\% \]

Alternatively, you can solve for the capital gains yield by first finding the expected price next year.
\[ N = 8, I/YR = 9.6911, PMT = 80, FV = 1000 \]
\[ PV = -908.76, V_B = 908.76 \]

Hence, the capital gains yield is the percentage price appreciation over the next year.
\[ CGY = \frac{(P_1 - P_0)}{P_0} = \frac{(908.76 - 901.40)}{901.40} = 0.816\% \]

c. As rates change they will cause the end-of-year price to change and thus the realized capital gains yield to change. As a result, the realized return to investors will differ from the YTM.

7-11  
\[ a. \text{ Using a financial calculator, input the following to solve for YTM:} \]
\[ N = 18, PV = -1100, PMT = 60, FV = 1000, \text{ and solve for } YTM = I/YR = 5.1355\% \]

However, this is a periodic rate. The nominal \( YTM = 5.1355\%(2) = 10.2709\% \approx 10.27\% \).

For the YTC, input the following:
\[ N = 8, PV = -1100, PMT = 60, FV = 1060, \text{ and solve for } YTC = I/YR = 5.0748\% \]

However, this is a periodic rate. The nominal \( YTC = 5.0748\%(2) = 10.1495\% \approx 10.15\% \).
So the bond is likely to be called, and investors are most likely to earn a 10.15% yield.

**b.** The current yield = $120/$1,100 = 10.91%. The current yield will remain the same; however, if the bond is called the YTC reflects the total return (rather than the YTM) so the capital gains yield will be different.

**c.**

\[
\text{YTM} = \text{Current yield} + \text{Capital gains (loss) yield} \\
10.27\% = 10.91\% + \text{Capital loss yield} \\
-0.64\% = \text{Capital loss yield.}
\]

This is the capital loss yield if the YTM is expected.

However, based on our calculations in Part a the total return expected would actually be the YTC = 10.15%. So, the expected capital loss yield = 10.15% - 10.91% = -0.76%.

**7-12 a.** Yield to maturity (YTM):

With a financial calculator, input \( N = 28, PV = -1165.75, PMT = 95, FV = 1000, I/YR = ? \) \( I/YR = \text{YTM} = 8.00\% \).

**Yield to call (YTC):**

With a calculator, input \( N = 3, PV = -1165.75, PMT = 95, FV = 1090, I/YR = ? \) \( I/YR = \text{YTC} = 6.11\% \).

**b.** Knowledgeable investors would expect the return to be closer to 6.1% than to 8%. If interest rates remain substantially lower than 9.5%, the company can be expected to call the issue at the call date and to refund it with an issue having a coupon rate lower than 9.5%.

**c.** If the bond had sold at a discount, this would imply that current interest rates are above the coupon rate (i.e., interest rates have risen). Therefore, the company would not call the bonds, so the YTM would be more relevant than the YTC.

**7-13** The problem asks you to solve for the YTM and Price, given the following facts:

\( N = 5 \times 2 = 10, \text{PMT} = 80/2 = 40, \text{and } FV = 1000. \) In order to solve for \( I/YR \) we need \( PV \).

However, you are also given that the current yield is equal to 8.21%. Given this information, we can find \( PV \) (Price).

Current yield = Annual interest/Current price
\[
0.0821 = \frac{80}{PV} \\
80 = PV / 0.0821 = 974.42.
\]

Now, solve for the YTM with a financial calculator:

\( N = 10, PV = -974.42, PMT = 40, \text{and } FV = 1000. \) Solve for \( I/YR = \text{YTM} = 4.32\% \). However, this is a periodic rate so the nominal YTM = 4.32\%(2) = 8.64\%.