Chapter 2: Sets, Whole Numbers, and Functions

2.4 Multiplication and Division of Whole Numbers

2.4.1. Vocabulary

2.4.1.1. array – any arrangement of objects into rows and columns – a $5 \times 4$ array contains 20 objects arranged in 5 rows and 4 columns

2.4.1.2. product – the result from multiplying two factors – factor $\times$ factor $=$ product

2.4.1.3. factor – any number used to multiply another number (also a factor) to obtain a product – factor $\times$ factor $=$ product

2.4.1.4. dividend – the number under the vinculum being divided into parts – the product in this model: factor $\times$ missing factor $=$ product

2.4.1.5. divisor – the number being divided into the dividend or the known factor in the model: factor $\times$ missing factor $=$ product

2.4.1.6. quotient – the number of times the divisor can divide into the dividend or the missing factor in this model: factor $\times$ missing factor $=$ product

2.4.1.7. missing factor - quotient

2.4.1.8. remainder – the fractional portion of a quotient

2.4.1.9. division algorithm – $a = bq + r$ with $0 \leq r < b$

2.4.2. Multiplication of Whole Numbers

2.4.2.1. Multiplication facts

2.4.2.1.1. digit $\times$ digit $=$ product

2.4.2.1.2. factor $\times$ factor $=$ product

2.4.2.1.3. 100 multiplication facts

2.4.2.2. multiplication fact families

Product | Multiplication facts that generate the given product
--- | ---
0 | $0 \times 0 \ 1 \times 0 \ 2 \times 0 \ 3 \times 0 \ldots \ 0 \times 6 \ 0 \times 7 \ 0 \times 8 \ 0 \times 9$ (19 facts)
1 | $1 \times 1$
2 | $1 \times 2 \ 2 \times 1$
\ldots
12 | $2 \times 6 \ 3 \times 4 \ 4 \times 3 \ 6 \times 2$

and so on

2.4.2.3. Repeated-Addition Model

2.4.2.3.1. $4 \times 5 = 4$ sets OF $5 = 5 + 5 + 5 + 5 = 20$

2.4.2.3.2. $5 \times 4 = 5$ sets OF $4 = 4 + 4 + 4 + 4 + 4 = 20$

2.4.2.4. The Array and Area Models

2.4.2.4.1. $4 \times 5$ array with 4 rows of 5 columns

2.4.2.4.2. $5 \times 4$ array with 5 rows of 4 columns

2.4.2.4.3. rows always first, rows are horizontal

2.4.2.4.4. columns always second, columns are vertical
2.4.2.5. **Definition of Multiplication of Whole Numbers**: For any whole numbers \(a\) and \(n \neq 0\),
\[ n \cdot a = a + a + a + \cdots + a \] If \(n = 0\), then \(0 \cdot a = 0\)

2.4.2.6. **Cartesian-Product Model**
- book assumes ANYONE would willingly order a soyburger! 😒
- makes use of tree diagram
- makes use of the notion of cross products
  2.4.2.6.3.1. Hotdog buns: \(B = \{\text{wheat, potato}\}; \ C = \{\text{ketchup, mustard, relish}\}\)
  2.4.2.6.3.2. \(B \times C = \{\text{wk, wm, wr, pk, pm, pr}\}\)

2.4.2.7. **Alternative Definition of Multiplication of Whole Numbers**: For finite sets \(A\) and \(B\), if \(n(A) = a\) and \(n(B) = b\), then \(a \cdot b = n(A \times B)\)

2.4.2.8. **Now try this 2-17 p. 114**: Work in your groups

2.4.3. **Properties of Whole-Number Multiplication**
- **Properties of Multiplication of Whole Numbers**
  2.4.3.1.1. **Closure property of multiplication of whole numbers** – For any whole numbers \(a\) and \(b\), \(a \cdot b\) is a unique whole number
  2.4.3.1.2. **Commutative property of multiplication of whole numbers** – For any whole numbers \(a\) and \(b\), \(a \cdot b = b \cdot a\)
  2.4.3.1.3. **Associative property of multiplication of whole numbers** – For any whole numbers \(a\), \(b\), and \(c\), \((a \cdot b) \cdot c = a \cdot (b \cdot c)\)
  2.4.3.1.4. **Identity property of multiplication of whole numbers** – There is a unique whole number 1 such that for any whole number \(a\), \(a \cdot 1 = 1 \cdot a = a\)
  2.4.3.1.5. **Zero multiplication property of whole numbers** – For any whole number \(a\), \(a \cdot 0 = 0 \cdot a = 0\)

2.4.3.2. **The Distributive Property of Multiplication Over Addition**
- Property: The distributive property of multiplication over addition for whole numbers: For any whole numbers \(a\), \(b\), and \(c\), \(a(b + c) = ab + ac\)

- **Properties**
  2.4.3.4. \(a(b + c + d) = ab + ac + ad\)
  2.4.3.5. \((a + b)(c + d) = a(c + d) + b(c + d) = ac + ad + bc + bd\)

2.4.4. **Order of Operations**
- Picture Eleven Million Dollars Actually Spent! 😊
- Does your calculator understand the order of operations?
- Enter \(2 + 3 \times 6\) into your calculator
- What did you get?
  2.4.4.4.1. 20? Then calculator uses order of operations
  2.4.4.4.2. 30? Then calculator does NOT use order of operations

2.4.5. **Division of Whole Numbers**
- **Set (Partition) Model**
  2.4.5.1. partitioning product into equal valued sets
- **Missing-Factor Model**
  2.4.5.2.1. factor \(x\) missing factor = product
  2.4.5.2.2. if know how to multiply, know how to divide

- **Definition of Division of Whole Numbers**: For any whole numbers \(a\) and \(b\), with \(b \neq 0\), \(a \div b = c\) if, and only if, \(c\) is the unique whole number such that \(b \cdot c = a\)
- **Repeated-Subtraction Model**
  2.4.5.4.1. \(18 \div 6 = ?\)
  2.4.5.4.1.1. \(18 - 6 - 6 - 6 = 0\)
  2.4.5.4.2. \(7448892 \div 132 = ?\)
  2.4.5.4.2.1. \(7448892 - 132 - 132 - ???\) need a better method

2.4.6. **The Division Algorithm**
2.4.6.1. Given any whole numbers $a$ and $b$ with $b \neq 0$, there exist whole numbers $q$ (quotient) and $r$ (remainder) such that $a = bq + r$ with $0 \leq r < b$

2.4.7. Division by 0 and 1
2.4.7.1. Try these in your calculator:
   2.4.7.1.1. $3 \div 0 = ?$
   2.4.7.1.2. $0 \div 3 = ?$
   2.4.7.1.3. $0 \div 0 = ?$

2.4.7.2. Division involving zero (where $n$ is any number except zero)
   2.4.7.2.1. $n \div 0$ is UNDEFINED
   2.4.7.2.2. $0 \div n = 0$
   2.4.7.2.3. $0 \div 0$ is UNDEFINED; some times called indeterminate

2.4.7.3. $n \div 1 = n$
2.4.7.4. See picture bottom of page 122! ☺
2.4.7.5. **MUST** Memorize this: Dividing by zero is undefined.

2.4.8. Ongoing Assessment p. 124
2.4.8.1. Home work: 1b, 3ae, 4a, 6ac, 8b, 9ac, 17, 19, 23a, 26, 31bc