

# HIGH SCHOOL MATH TOURNAMENT

2016



## DIRECTIONS:

1. Do NOT open this exam booklet until you are told to do so.
2. All electronics, including calculators, must be OFF and out of sight. You may only use the scratch paper provided. (Raise hand if you need a second sheet.)

Solve as many of these problems as you can. All problems, except for the purpose of tie-breaking, are equally weighted. So it may be to your advantage to skip around.

**All answers must be clearly indicated on your ANSWER SHEET.**

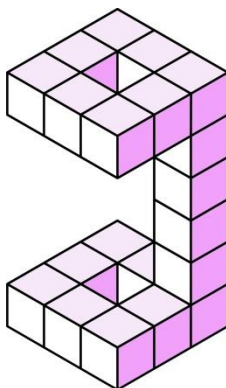
**Please print your name and the name of your school in full on your ANSWER SHEET.**

Anything written on the pages of the exam booklet or scratch paper will not be graded.

**However, feel free to write on the pages of the exam booklet, if it helps you solve the problems.** After you are finished, if you need a copy of this exam or a key to this exam, email Dr. Arsalan Wares (Awares@valdosta.edu).

You will get 60 minutes to work on the exam.

**PART I.** (Multiple Choice Questions) Choose the best answer. For each question in this section, **write your answers legibly on your answer sheet.** There is no penalty for wrong answers. Feel free to write anywhere on the pages this exam booklet.



**Figure 1.** For problems 1 and 2.

1. The solid shown in **figure 1** was made with unit cubes. The cubes are touching face-to-face and there are no hidden cubes. Find the volume the solid.

- |                  |                  |                  |
|------------------|------------------|------------------|
| a. 20 unit cubes | b. 22 unit cubes | c. 18 unit cubes |
| d. 16 unit cubes | e. 14 unit cubes | f. none of these |

2. The solid shown in **figure 1** was made with unit cubes. The cubes are touching face-to-face and there are no hidden cubes. Find the total surface area of the solid.

- |                    |                    |                    |
|--------------------|--------------------|--------------------|
| a. 20 unit squares | b. 80 unit squares | c. 88 unit squares |
| d. 84 unit squares | e. 78 unit squares | f. none of these   |

3. The sum of two numbers is 3 and the product of the two numbers is  $9/4$ . Find the sum of squares of the two numbers

- |           |          |                  |
|-----------|----------|------------------|
| a. 9      | b. $9/2$ | c. 5             |
| d. $11/2$ | e. 3     | f. none of these |

4. If  $f(x) = \frac{3x-1}{x+3}$ , then determine the inverse function,  $y = f^{-1}(x)$ .

- |                       |                       |                    |
|-----------------------|-----------------------|--------------------|
| a. $\frac{x+3}{3x-1}$ | b. $\frac{3x-1}{x-3}$ | c. $\frac{x+3}{3}$ |
| d. $\frac{x-3}{3x+1}$ | e. $\frac{3x+1}{3-x}$ | f. none of these   |

5. If  $\log_{16} x = y$ , then find  $\log_2 x$ .

a.  $3y$

b.  $16y$

c.  $\frac{\log_2 x}{\log_2 8}$

d.  $4y$

e.  $\frac{y}{\log_2 8}$

f. none of these

6. Find the solution to  $|x - 5| > 3$ .

a.  $(8, \infty)$

b.  $(-3, \infty)$

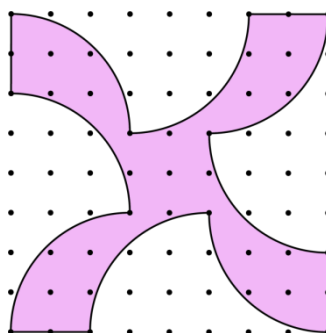
c.  $(-\infty, 8)$

d.  $(-\infty, 2) \cup (8, \infty)$

e.  $(2, 8)$

f.  $(-\infty, -2) \cup (8, \infty)$

7. In **figure 2**, the shortest distance between two adjacent dots is 1 unit, and the shaded region is bounded by 8 quarter circular arcs and 4 line segments. Find the area of the shaded region.



**Figure 2.** For problem 7.

a. 20 unit squares

b. 26 unit squares

c. 30 unit squares

d. 24 unit squares

e. 28 unit squares

f. cannot be determined

8. Carl is a bibliophile. Last year the ratio of the number of mathematics books to the number of physics books in his collection was 7:5. This year he purchased 13 more mathematics books and donated 25 of the physics books to the local library. Now he has 3 times as many mathematics books as physics books. How many mathematics books does he have in his collection now?

a. 90

b. 11

c. 77

d. 95

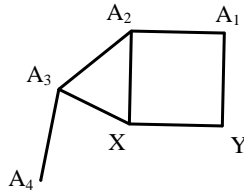
e. 100

f. none of these

9. A box contains 25 coins, only quarters and dimes. The amount of money in the box is \$4.45. How many quarters are in the box?

- a. 13
- b. 14
- c. 15
- d. 12
- e. 25
- f. none of these

10.  $A_1A_2\dots A_n$  is a polygon with  $n$  sides (see **figure 3**). Only a part of  $A_1A_2\dots A_n$  is shown in **figure 3**. If polygon  $A_1A_2XY$  is a square, and the measure of  $\angle A_4A_3X$  is 10 degrees more than that of  $\angle A_2A_3X$ , find the value of  $n$ .



**Figure 3.** For Problem 10

- a. 7
- b. 8
- c. 9
- d. 10
- e. 11
- f. none of these

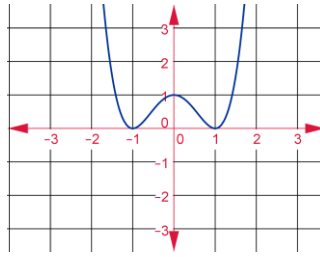
11. You discover that  $\frac{2}{5}$  of your savings is enough to buy  $\frac{3}{4}$  of the candy in the jar. What part of your savings will be left after you purchase the entire jar of candy?

- a.  $\frac{11}{20}$
- b.  $\frac{8}{15}$
- c.  $\frac{7}{15}$
- d.  $\frac{13}{20}$
- e.  $\frac{9}{15}$
- f. none of these

12. The arithmetic mean of the first set of 6 quiz scores is 60. The arithmetic mean of the second set of 9 quiz scores is 75. If all 15 quizzes are weighed the same, what is the arithmetic mean of the 15 quiz scores?

- a. 62
- b. 65
- c. 67
- d. 68
- e. 69
- f. none of these

13. **Figure 4** shows a sketch of the following function:  $g(x) = [f(x)-1]^2$ . What could be the best possible sketch of  $y = f(x)$ ?



**Figure 4.** For Problem 13

- a. b. c.
- d. e. f. none of these

14. The first bag has 4 red marbles and 2 green marbles. The second bag has 3 red marbles and 2 green marbles. One of the bags is randomly selected, and then one marble is selected from that bag without looking. Assume the bags are identical and you cannot see the content of the bags. What is the probability that you will get a red marble?

- a. 11/30                      b. 19/30                      c. 2/3  
d. 12/30                      e. 18/30                      f. none of these

15. Consider the following two sets:

$$A = \{f, l, o, r, i, d, a\} \text{ and } B = \{i, d, a, h, o\}.$$

If you randomly pick a letter from set  $A$ , what is the probability that it will be in set  $B$ ?

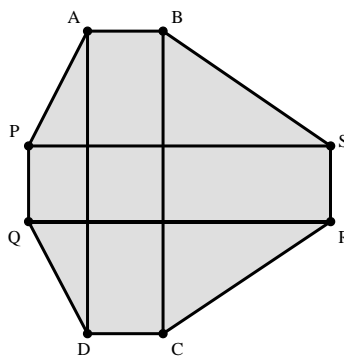
- a. 4/5                      b. 4/7                      c. 2/7  
d. 0                      e. 1                      f. none of these

PART II (Open-ended Questions). These problems vary in difficulty. Questions are not arranged by difficulty. Give exact answers whenever possible and follow the usual conventions for simplifying algebraic and numerical expressions. There is no penalty for wrong answers. For each question in this section, **write your answers legibly on your answer sheet**. Feel free to write anywhere on the pages this exam booklet.

16. Jeff has 2 times as many stamps as John. How many stamps must Jeff give John so that they will each have 120 stamps?

Enter answer on exam sheet

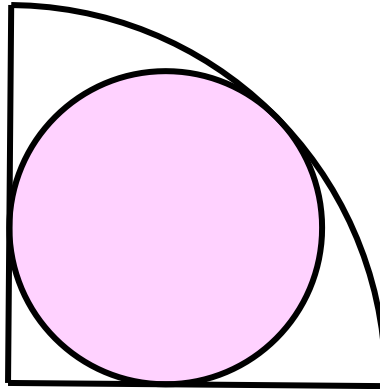
17. In **figure 5**, Polygons  $ABCD$  and  $PQRS$  are congruent rectangles, each with dimensions 8 cm by 2 cm. Rectangles  $ABCD$  and  $PQRS$  intersect each other as shown at right angles. Find the area of octagon  $ABSRCDQP$ .



**Figure 5.** For Problem 17

Enter answer on exam sheet

18. **Figure 6** shows a quarter sector with a circle inside it. The circle is shaded and this shaded circle is touching the radii and arc of the quarter sector. Find the ratio of the area of the circle to the area of the quarter sector. Note a sector is a part of the circle that is bounded by an arc and two radii.



**Figure 6.** For Problem 18

Enter answer on exam sheet

19. Mr. and Mrs. Yakov are going to have a baby. The baby's last name will be Yakov as well. The parents want their baby's monogram (first, middle and last initials) to be in alphabetical order with no letters repeated. How many such monograms are possible?

Enter answer on exam sheet

20. A survey of 100 American tourists in Europe was conducted and the following information was gathered:

15 have been to Amsterdam

23 have been to Berlin

33 have been to Copenhagen

6 have been to Amsterdam and Berlin

7 have been to Amsterdam and Copenhagen

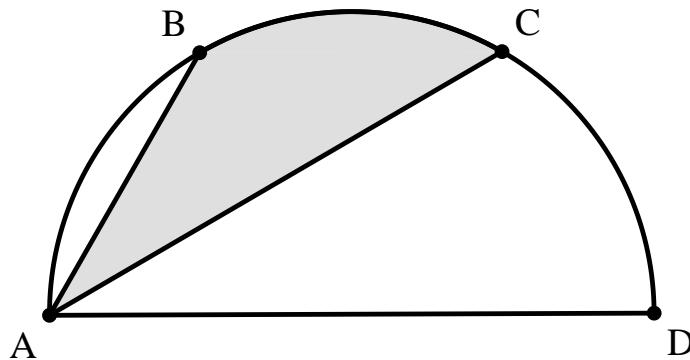
9 have been to Berlin and Copenhagen

16 have been to exactly two of the following three cities: Amsterdam, Berlin and Copenhagen

How many tourists have not been to any of the three cities: Amsterdam, Berlin and Copenhagen?

Enter answer on exam sheet

21. **Figure 7** shows a semicircle with its diameter along line segment  $AD$ . Points  $B$  and  $C$  trisect the semicircular arc. Part of the semicircle bounded by line segments  $AB$ ,  $AC$  and arc  $BC$  is shaded. If the radius of the semicircle is 6, find the area of the shaded region.



**Figure 7.** For Problem 21

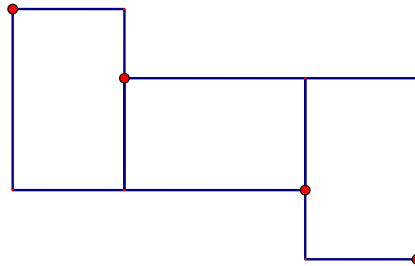
Enter answer on exam sheet



22. It takes Johnny and his dad 5 hours to wash the family car. Kate, Johnny's sister, can wash the car 3 times as fast as Johnny. When Johnny is not available, it takes Kate and her dad 4 hours to wash the family car. How long would it take Kate and Johnny, all by themselves, to wash the family car?

Enter answer on exam sheet

23. The octagonal shape in **figure 8** was constructed with three congruent  $a$  by  $b$  rectangles, where  $a$  and  $b$  are real numbers, and  $a < b$ . Four of the vertices of the three rectangles are marked with large dots. If these four vertices are collinear, find the ratio  $a:b$ .



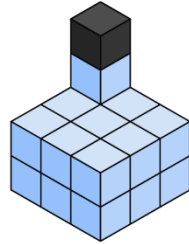
**Figure 8.** For Problem 23

Enter answer on exam sheet

24. Jeremy drives from his hotel to the airport to catch a flight. He drives 35 miles in the first hour, but realizes that he will be 1 hour late if he continues at this speed. He increases his speed by 15 miles per hour for the rest of the way to the airport and arrives 30 minutes early. Assume it takes negligible time to increase his speed, how many miles is the airport from his hotel?

Enter answer on exam sheet

25. The solid shown in **figure 9** is made with 20 cubes of the same size. One of the 20 cubes is dark in color. When the dark cube is carefully removed, the surface area of the solid formed by the remaining 19 cubes is 64 square inches less than the surface area of the solid shown in **figure 9**. Find the volume of the solid shown in **figure 9**.

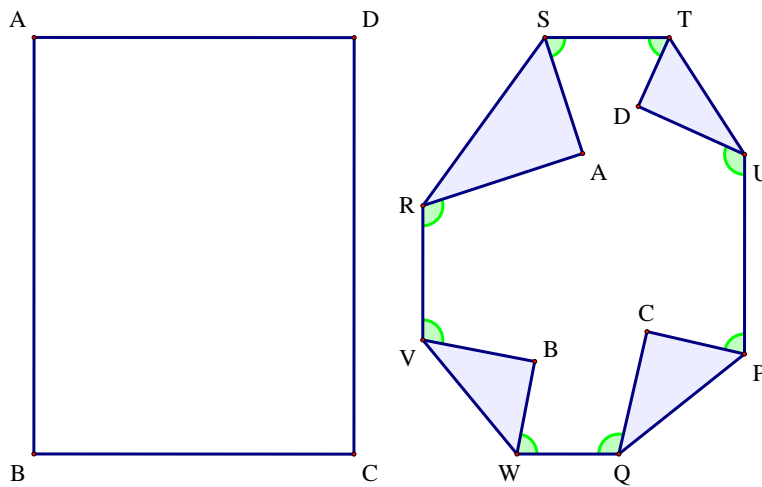


**Figure 9.** For Problem 25

Enter answer on exam sheet

26. A rectangular sheet of paper was folded four times as shown in **figure 10**. Find the following sum of angles:

$$m\angle ARV + m\angle RVB + m\angle BWQ + m\angle WQC + m\angle CPU + m\angle PUD + m\angle DTS + m\angle TSA.$$



**Figure 10.** For Problem 26

Enter answer on exam sheet

2016 VSU High School Contest

PART I. Multiple Choice

1.	a	6.	d	11.	c
2.	e	7.	e	12.	e
3.	b	8.	a	13.	b
4.	e	9.	a	14.	b
5.	d	10.	c	15.	b

PART II. Free Response

16.	40	22.	10 hours
17.	46 sq cm	23.	$\frac{(\sqrt{5}-1)}{2}$
18.	$\frac{4}{3+2\sqrt{2}}=4(3-2\sqrt{2})$	24.	210 miles
19.	276	25.	1280 cubic inches
20.	49	26.	720°
21.	$\frac{1}{6}\pi(6^2)=6\pi$		